Modelling Spot FX Market

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Introduction to the currency markets

Foreign exchange market

- The foreign exchange (FX) market is a global decentralized or over-the-counter (OTC) market for the trading of currencies.
- This market determines the foreign exchange rate. It includes all aspects of buying, selling and exchanging currencies at current or determined prices.
- In terms of trading volume, it is by far the largest market in the world.

Market role

- The foreign exchange market assists international trade and investments by enabling currency conversion.
- It permits a business in the United States to import goods from Eurozone members, and pay Euros, even though its income is in United States dollars.
- It also allows direct speculation and evaluation relative to the value of currencies and the carry trade speculation.

Market size

- According to the Bank for International Settlements, the preliminary global results from the 2016 Triennial Central Bank Survey of Foreign Exchange and OTC Derivatives Markets Activity show that trading in foreign exchange markets averaged $5.09 trillion per day in April 2016.
- For comparison GDP of USA in 2017 was $19.39 trillion.

Market split

The $5.09 trillion break-down is as follows:

- $1.654 trillion in spot transactions
- $700 billion in outright forwards
- $2.383 trillion in foreign exchange swaps
- $96 billion currency swaps
- $254 billion in options and other products

Market participants

- The main participants in this market are the larger international banks.
- Financial centres around the world facilitate trading between a wide range of multiple types of buyers and seller.
- Currencies are always traded in pairs, the foreign exchange market determines their relative value by setting the market price of one currency if paid for with another. Ex: 1 USD is worth X CAD, or CHF, or JPY, etc..

ISO 4217 code

- Each currency pair thus constitutes an individual trading product and is traditionally noted XXXYYY, where XXX and YYY are the ISO 4217 international three-letter code of the currencies involved.
- The first currency (XXX) is the base currency that is quoted relative to the second currency (YYY), called the counter currency (quote or reference currency).

Market convention

- The market convention is to quote most exchange rates against the USD with the US dollar as the base currency (e.g. USDJPY, USDCAD, USDCHF).
- The exceptions are the British pound (GBP), Australian dollar (AUD), the New Zealand dollar (NZD) and the euro (EUR), where the USD is the counter currency (e.g. GBPUSD, AUDUSD, NZDUSD, EURUSD).
- For instance, the quotation EURUSD 1.23346 is the price of the Euro expressed in US dollars, meaning 1 euro = 1.23346 dollars.

Spot market

- According to the 2016 Triennial Survey, the most heavily traded bilateral currency pairs were:
- EURUSD: 23.0%
- USDJPY: 17.7%
- GBPUSD (also called cable): 9.2%
- The U.S. currency was involved in 87.6% of transactions, followed by the euro (31.3%), the yen (21.6%), and sterling (12.8%).
- Volume percentages for all individual currencies should add up to 200%, as each transaction involves two currencies.

Most traded currencies by value – total 200%. Top 10 – 178.5%

Rank	Currency	ISO 4217 code (symbol)	% daily share (April 2016)
1	United States dollar	USD ($)	87.6%
2	Euro	EUR (€)	31.4%
3	Japanese yen	JPY (¥)	21.6%
4	Pound sterling	GBP (£)	12.8%
5	Australian dollar	AUD (A$)	6.9%
6	Canadian dollar	CAD (C$)	5.1%
7	Swiss franc	CHF (Fr)	4.8%
8	Renminbi	CNY (元)	4.0%
9	Swedish krona	SEK (kr)	2.2%
10	New Zealand dollar	NZD (NZ$)	2.1%

Trading centres

- The main trading centres are London and New York, though Tokyo, Hong Kong and Singapore are important centres as well.
- Currency trading happens continuously throughout the day; as the Asian trading session ends, the European session begins, followed by the North American session and then back to the Asian session.
- There is no centrally cleared market, and there is very little cross-border regulation.

Trading platforms

- Due to London's dominance in the market, a particular currency's quoted price is usually the London market price.
- Major trading exchanges include Electronic Broking Services (EBS) and Thomson Reuters Dealing.
- Major banks also offer trading systems.
- A joint venture of the Chicago Mercantile Exchange and Reuters, called Fxmarketspace opened in 2007 and aspired but failed to the role of a central market clearing mechanism.

Market share – Top 10 have 66.9%

Top 10 currency traders % of overall volume, May 2016		
Rank	**Name**	**Market share**
1	Citi	12.9 %
2	JP Morgan	8.8%
3	UBS	8.8%
4	Deutsche Bank	7.9%
5	Bank of America Merrill Lynch	6.4%
6	Barclays	5.7%
7	Goldman Sachs	4.7%
8	HSBC	4.6%
9	XTX Markets	3.9%
10	Morgan Stanley	3.2%

The model

Oomen model

- Consider the spot FX market in which there are only market makers and traders.
- Market makers, known also as liquidity providers, continually quote indicative bid (buy) and ask (sell) prices.
- Traders follow the market and send trade requests to the market makers based on the best bid and ask prices they see in their liquidity pool.

Oomen model cont.

- Different traders relay on different liquidity pools.
- Market makers receive streams of trade requests from the traders.
- They have the right of the last look and may reject or accept a trade.
- The model is defined in discrete time
- The model assumes a single trader and many market makers

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Oomen model in continuous time

Continuous time model

- The unobserved (logarithmic) price process of an exchange rate, say EURUSD, is given by

$$p(t) = p(0) + \sigma W(t); \quad t > 0$$

- $W(t), t > 0$ is a Brownian motion

- We assume there are many Market Makers

Bid and Ask prices

- The dynamics of the bid and ask prices of the market maker j are

$$b(j,t) = p(j,t) - \frac{s(j)}{2},$$

$$a(j,t) = p(j,t) + \frac{s(j)}{2}$$

- The process $p(j,t)$ is given by

$$p(j,t) = p(t) + m(j,t)$$

Process $m(j,t)$

- The process $m(j,t)$ is a zero mean continuous stationary Gauss Markov process with the covariance function

$$Em(j,t)m(j,s) = \omega(j)^2 e^{-\beta(j)|t-s|}$$

- The process $m(j,t)$ can, of course, be defined as a solution to a linear stochastic differential equation driven by a Brownian motion $B(j,t)$.

SDE based definition

- Indeed the process $m(j,t)$ can be constructed as the solution of the following stochastic differential equation

$$dm(\mathrm{j},\mathrm{t}) = -\beta_j m(j,t)dt + \sqrt{\beta_j}\omega_j dB(j,t)$$

- The random variable $m(j,0)$ is a $N(0,{\omega_j}^2)$ and is independent of the Brownian motion $B(j,t)$

Dependence structure

- The Brownian motions $B(j,t)$ are correlated with the correlations $\rho(i,j)$.
- The process $W(t)$ and $B(j,t)\ j = 1,2,3,\ldots$ are independent
- The vector $m(j,0), j = 1,2,3,\ldots$ and the process $B(j,t), j = 1,2,3,\ldots$ are independent
- The above model is a direct generalization to the continuous time of the discrete time model proposed by Oomen.

Trader perspective

Trader environment

- Unlike in the Oomen's model we assume there are many traders
- Trader j observes quotes published by several market makers
- Different traders relay on different pools of market makers to observe quoted prices
- In our framework traders may be corporates, asset managers, pension funds, insurance companies or hedge funds

Trader behaviour

- Assume initially that each trader deals in standard amounts on the best available price
- This means trader j will submit a request to buy to the market maker who quotes the minimal indicative price
- This also means that trader j will submit request to sell to the market maker who quotes the maximal indicative price

Trader best bid and best ask

- Assume Trader i observes quotes published by a group of Market Makers T_i
- The best bid of the Trader i is the highest price at which he can sell, which is

$$\max_{j \in T_i} b(j,t)$$

- The best ask of the Trader i is the lowest price at which he can buy, which is

$$\min_{j \in T_i} a(j,t)$$

Trade routing

- Different traders see different best bids and best asks
- Even if they act on their best bids and best asks they are likely to route their trades to different market makers
- The decision how to route the trades to the different market makers may be driven by many different of factors

Possible factors

- Counterparty credit limits
- Better understanding by a trader (hedge fund) how to arbitrage a market maker
- Better understanding of the unobserved true price

Market maker perspective

Market maker environment

- A trader will send trade request to a market maker
- The market maker will be able to accept or reject it (last look)
- The intensity with which trade orders will reach a market maker depends on how far is his bid and his ask from the best bids and the best asks the traders can see in their liquidity pools

Trade requests

- If we assume only one trader his requests will go to the market makers publishing the best bid and the best ask prices
- With many traders acting on different information, using different market makers, and having better or worse understanding of the fundamental price dynamics an individual market maker is likely to receive trade requests coming from different traders

Marker maker aggregator

- We aggregate the behaviour of different traders into description of flow of trade requests market maker j receives
- We assume they are represented by a Poisson process with intensity for ‘ask trades’ given by

$$\mu^a\big(a(j,t) - a(t)\big)$$

- The process $a(t)$ represents the unobserved market ask prices. The function μ^a is also unknown and will be estimated from the data

Market maker aggregator cont.

- The ‘bid trade’ requests market maker j receives follow a Poisson process with intensity

$$\mu^b(b(t) - b(j,t))$$

- The process $b(t)$ represents the unobserved market bid prices. The function μ^b needs to be estimated from the data
- We further assume that the processes describing arrivals of ‘ask’ and ‘bid’ trade requests are independent

Flow of trade requests

- Flow of all trade requests market maker j receives is a Poisson process with intensity

$$\mu^a\big(a(j,t) - a(t)\big) + \mu^b(b(t) - b(j,t))$$

- Note that functions $\mu^a(\mathrm{x})$ and $\mu^b(x)$ are independent of j and hence are market specific

Intensity of flow requests

- The closer the indicative ask and bid prices of the market maker j are to the market ask and bid, respectively, the more trade requests will arrive
- The market ask and bid prices may differ from the best ask and the best bid a trader sees in his liquidity pool
- They represent an aggregate of the best bid and ask prices all traders see in their liquidity pools

Last look

- Traders have information advantage over market makers because they source indicative quotes from many market makers and may act on the best available price
- Last look is a process which allows the market maker to decide whether to accept – and if so at what rate – or reject a trader's deal request

Last look as an option

- Generally the trade is rejected if in a fixed period of time the trade moves against the market maker beyond some threshold
- The market makers have in fact an option of withdrawing the price they made to the market
- This option of course has a value
- There are many different designs of the last look protocol

Last look valuation

- The standard replication argument does not apply
- Market maker may quote spreads to the market so that his expected profits are zero
- Inventory management imposes additional constraints on the last look protocol design
- In this context indifference based valuation principle may be used

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